

Baryogenesis from Mixing of Lepton Doublets

Björn Garbrecht

*Physik Department T70, James-Franck-Straße,
Technische Universität München, 85748 Garching, Germany
and Institut für Theoretische Teilchenphysik und Kosmologie,
RWTH Aachen University, 52056 Aachen, Germany*

Abstract

It is shown that the mixing of lepton doublets of the Standard Model can yield sizeable contributions to the lepton asymmetry, that is generated through the decays of right-handed neutrinos at finite temperature in the early Universe. When calculating the flavour-mixing correlations, we account for the effects of Yukawa as well as of gauge interactions. We compare the freeze-out asymmetry from lepton-doublet mixing to the standard contributions from the mixing and direct decays of right-handed neutrinos. The asymmetry from lepton mixing is considerably large when the mass ratio between the right-handed neutrinos is of order of a few, while it becomes Maxwell-suppressed for larger hierarchies. For an intermediate range between the case of degenerate right-handed neutrinos (resonant Leptogenesis) and the hierarchical case, lepton mixing can yield the main contribution to the lepton asymmetry.

1 Introduction

Sources for CP -violating effects are often categorised into contributions from mixing and from direct decays. This applies to Leptogenesis [1] as well, where usually the mixing [2] and the direct decays of right-handed singlet neutrinos N_i are accounted for. Of particular interest is the source from mixing, because in the situation where the mass-difference of the right-handed neutrinos is small, it gives rise to a resonantly enhanced CP -asymmetry [2–10].

Besides the right-handed neutrinos, Higgs bosons and Standard Model lepton doublets directly take part in Leptogenesis. Direct decay asymmetries from Higgs bosons and lepton doublets are usually not considered, because in the vacuum, these particles cannot decay into heavy right-handed neutrinos. Consequently, at zero temperature,

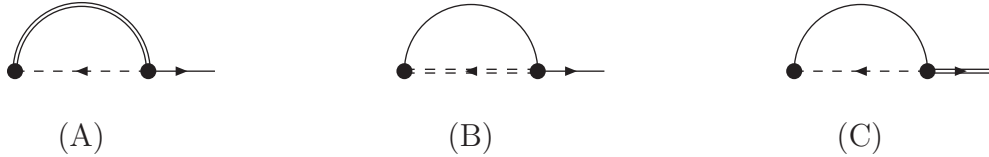


Figure 1: Diagrammatic representation of contributions to the collision term that yield a source for the lepton asymmetry (A) through mixing of right-handed neutrinos, (B) through mixing of Higgs doublets and (C) through mixing of lepton doublets. Solid lines with arrows are propagators for the Standard Model lepton doublets ℓ , dashed lines with arrows are propagators for Higgs doublets ϕ and solid lines without arrows are propagators for the right-handed neutrinos N . The double lines represent resummed propagators that in particular account for the mixing of flavour.

there are no kinematic cuts through the propagators of a right-handed neutrino and a lepton- or Higgs-doublet that can lead to CP -violation. At finite temperature however, these cuts contribute to the asymmetry [11–13], since the plasma continuously absorbs and produces leptons and Higgs bosons.

Concerning mixing, right-handed neutrinos, as these are sterile, are well suited for the efficient production of asymmetries since flavour off-diagonal (where we define the flavour of these particles by their mass-eigenstates, in contrast to Standard Model leptons) correlations are only damped by the Yukawa couplings, which are small by the requirement of a substantial deviation of the right-handed neutrinos from equilibrium.

Gauge interactions, in particular in the Standard Model, are typically much larger than the Yukawa couplings of the right-handed neutrinos. However, as these are flavour blind, they do not directly damp off-diagonal flavour correlations. Therefore, also the gauged particles within the Standard Model or its extensions can lead to substantial CP -violation from mixing. For multiple Higgs doublets, it is shown in Ref. [14], that their mixing can be a viable source for Leptogenesis. In the present work, we show that the mixing of the lepton doublets of the Standard Model contributes a sizeable amount of CP -violation to Leptogenesis as well.

In order to calculate the generation of the asymmetries in the finite temperature background, we use the Closed-Time-Path (CTP) method [8–10, 13–31]. The diagrammatic representation of the source terms for the asymmetry in Figure 1 indicates how the mixing of the various species yields contributions to Leptogenesis. When we assume that the right-handed neutrinos are heavier than the lepton doublets and Higgs bosons, the imaginary parts of the self-energies that appear within the resummed propagators in Figure 1(B) and 1(C) are purely thermal, as the corresponding $1 \leftrightarrow 2$ processes are kinematically forbidden in the vacuum. This resembles the situation for Leptogenesis from the mixing of right-handed neutrinos with masses of the GeV scale or below [10, 32–35], where the CP -violating cut is dominated by purely thermal contributions [10]. For the direct CP -violation, the contribution of such non-standard cuts is calculated in Ref. [13].

In Section 2, we derive the off-diagonal correlations within the distributions of the

left-handed lepton doublets. Even though gauge interactions do not directly violate flavour and damp its correlations, they catalyse the decay of flavour correlations in the presence of flavour-dependent masses or interactions. For flavoured Leptogenesis, the relevance of such effects is discussed in Ref. [27]. Here, we show that gauge interactions effectively limit the maximal resonant enhancement of a CP -asymmetry resulting from the mixing of gauged particles. For the mixing of Higgs doublets, this aspect is not discussed in Ref. [14], a shortcoming that is addressed in Section 5.

An important aspect within the present context is the role of lepton-number violation and conservation. The spinor trace of diagram 1(c), that quantifies the rate of CP -violation from mixing of lepton doublets, contains no lepton-number violation that could be described by insertions of an odd number of Majorana mass-terms. Therefore, lepton number must in some sense be conserved. This, we show in Section 3, where it turns out that the source for the asymmetry in the lepton doublets cancels the helicity asymmetry within the right-handed neutrinos. The latter is however rapidly violated by the Majorana masses, provided the right-handed neutrinos are non-relativistic, as it is the case in the strong washout regime.

In Section 4, we then calculate the freeze-out value of the asymmetry that results from mixing lepton doublets in the strong washout regime. This, we compare to the asymmetry from the standard sources, *i.e.* from the mixing and direct decays of right-handed neutrinos. For illustrative purposes, we choose a particular slice in parameter space and vary the mass ratio of the right-handed neutrinos between one (degenerate case) and a few. It is a characteristic feature of the asymmetry from lepton mixing, that it is exponentially small for large mass ratios, as the CP -violating cut is purely thermal and becomes Maxwell suppressed in that situation.

We close with summarising and concluding remarks, that are given in Section 6.

2 Off-Diagonal Correlations of Lepton Doublets

The model that we consider here is the same that underlies flavoured Leptogenesis [27, 36–38] and is given by the Lagrangian

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \bar{\psi}_{Ni} (i \not{\partial} - M_{Nij}) \psi_{Nj} + \bar{\psi}_{\ell a} i \not{\partial} \psi_{\ell a} + (\partial^\mu \phi^\dagger) (\partial_\mu \phi) \\ & - Y_{ia}^* \bar{\psi}_{\ell a} \tilde{\phi} P_R \psi_{Ni} - Y_{ia} \bar{\psi}_{Ni} P_L \tilde{\phi}^\dagger \psi_{\ell a} - h_{ab} \phi^\dagger \bar{\psi}_{Ra} P_L \psi_{\ell b} - h_{ab}^* \phi \bar{\psi}_{\ell b} P_R \psi_{Ra} . \end{aligned} \quad (1)$$

The Standard Model Higgs doublet is denoted by ϕ , the lepton doublets by ℓ_a , right-handed charged leptons by R_a and right-handed neutrinos by N_i . The associated spinors are denoted by ψ with the subscript of the respective field. We use the definition $\tilde{\phi}_a = \phi_b^* \epsilon_{ba}$, but suppress explicit indices of the $SU(2)_L$ gauge group in the following. We transform the N_i to a basis where M_N is diagonal and write $M_{Ni} = M_{Nii}$. Furthermore, we take h_{ab} to be diagonal, where $a, b = e, \mu, \tau$. We are mainly interested in the dynamics within the temperature range $10^9 \text{ GeV} < T < 10^{12} \text{ GeV}$, such that only the τ -lepton Yukawa couplings are in equilibrium. In that case, we can neglect h_{ee} and $h_{\mu\mu}$ and then

perform a unitary transformation of the lepton flavours ℓ_e and ℓ_μ to $\ell_{\sigma\perp}$ and ℓ_σ such that $Y_{2\sigma\perp} = Y_{3\sigma\perp} = 0$, see Ref. [27] for the explicit construction of such a transformation. (After this transformation, h is no longer diagonal within the block of the flavours e and τ . As we may however neglect these Yukawa couplings at temperatures above 10^9 GeV, h can still be considered as effectively diagonal). Then, only lepton flavour asymmetries within ℓ_τ and ℓ_σ are produced, while there is no asymmetry in the remaining linear combination $\ell_{\sigma\perp}$, since $\ell_{\sigma\perp}$ couples only to one of the right-handed neutrinos, and no CP -violating interference terms occur. While it is the salient feature of flavoured Leptogenesis that all CP -odd off-diagonal correlations between the flavours τ and σ are rapidly erased, because the interactions mediated by $h_{\tau\tau}$ are in equilibrium, we find in the present work that the deviation of the right-handed neutrinos from equilibrium does generically sustain CP -even [*cf.* Eq. (22) below] off-diagonal correlations of the leptons [Eq. (17) below], which in turn enter the source term for the CP -violating lepton asymmetry. We also note that the present discussion does not directly apply but can easily be generalised to temperatures below 10^9 GeV, where the flavours e and μ are distinguishable.

The present calculations based on the CTP formalism require numerous definitions that can be found within Refs. [8, 26, 27]. In particular $iS^{<, >}$ denotes fermionic and $i\Delta^{<, >}$ scalar Wightman functions. Subscripts on these indicate the particular fields and flavour correlations. Distribution functions for particles and anti-particles are denoted by f and \bar{f} , number densities by n and \bar{n} . Again, fields and flavour correlations are indicated by subscripts. A prefix δ implies that the particular quantity is the deviation from the equilibrium propagator, distribution function or density.

The dynamics of the out-of equilibrium distributions of left-handed lepton doublets can be described by the kinetic equation [27]

$$i\partial_\eta g_{hab}^{<, >} + h \left(\varsigma_{aa}^\# - \varsigma_{bb}^\# \right) g_{hab}^{<, >} = -\frac{1}{4} \text{tr} \left[i\mathcal{C}_\ell + i\mathcal{C}_\ell^\dagger \right]_{ab}, \quad (2)$$

where η is the conformal time in the expanding Universe. The functions $g_h^{<, >}$ appear within the spinor components of the left-handed lepton propagator $S_\ell^{<, >}$ and factorise into statistical and spectral terms in the narrow width limit [8, 18, 19, 39, 40]. When $|\mathbf{k}| \gg \sqrt{G}T$, where $G = (3/2)g_2^2 + (1/2)g_1^2$, we may approximate

$$g_{-ab}^{<, >}(k) = -2\pi\delta(k^2)|\mathbf{k}|\vartheta(k^0)\delta f_{\ell ab}(\mathbf{k}), \quad (3a)$$

$$g_{+ab}^{<, >}(k) = 2\pi\delta(k^2)|\mathbf{k}|\vartheta(-k^0)\delta \bar{f}_{\ell ab}(\mathbf{k}). \quad (3b)$$

For $|\mathbf{k}| \ll \sqrt{G}T$, this approximation, in particular the relation between the sign of k^0 and the helicity, becomes inaccurate due to the presence of hole modes. However, this region of soft momenta covers only a small part of the phase-space, such that the approximations (3) are sufficiently accurate for the present purposes. The flavour-dependent self-mass terms $\varsigma^\#$ are given by

$$\varsigma_{ab}^\#(k) = \frac{[h^\dagger h]_{ab}T^2}{16|\mathbf{k}|}, \quad (4)$$

where we have neglected the contributions from the couplings Y .

It is useful to decompose the collision term as

$$\mathcal{C}_\ell = \mathcal{C}_\ell^{CPV} + \mathcal{C}_\ell^Y + \mathcal{C}_\ell^h + \mathcal{C}_\ell^g \quad (5)$$

into contributions mediated by the Yukawa couplings Y and h and the gauge couplings g . Within \mathcal{C}_ℓ^Y , we collect the CP -conserving processes mediated by Y , which are to leading approximation quadratic in Y . The term \mathcal{C}_ℓ^{CPV} is mediated by Y as well and accounts for CP -violating processes. To leading approximation, it is fourth order in Y .

The part of the lepton self-energy mediated by Y is

$$i\Sigma_{\ell ab}^{Yfg}(k) = Y_{ai}^\dagger Y_{jb} \int \frac{d^4 p}{(2\pi)^4} P_R iS_{Nij}^{fg}(p) i\Delta_\phi^{gf}(p-k), \quad (6)$$

where f, g are CTP indices. The term \mathcal{C}_ℓ^Y is of importance for two reasons: First, as in the standard scenario of Leptogenesis, its diagonal components describe the washout of the CP -odd lepton asymmetries accumulated in the diagonal correlations of lepton flavours. (For flavoured Leptogenesis, the off-diagonal correlations are rapidly erased due to the Yukawa coupling $h_{\tau\tau}$.) Second, its off-diagonal components correspond to the source of CP -even, flavour off-diagonal correlations in the lepton doublets. As the CP -odd off-diagonal correlations are suppressed by the Yukawa interactions $h_{\tau\tau}$, and the CP -even off-diagonal correlations are suppressed by an order of Y^2 times the deviation of the right-handed neutrinos from equilibrium, when compared to the diagonal distribution functions, we can substitute the flavour diagonal components of $iS_\ell^{<,>}$ for the purpose of calculating the contributions to \mathcal{C}_ℓ^Y that are of leading importance. We thus obtain

$$\begin{aligned} \mathcal{C}_{\ell ab}^Y + \mathcal{C}_{\ell ab}^{CPV} &= i\Sigma_{\ell ac}^{Y>}(k) iS_{\ell cb}^{<}(k) - i\Sigma_{\ell ac}^{Y<}(k) iS_{\ell cb}^{>}(k) \\ &\approx -[Y^\dagger Y]_{ab} P_R \int \frac{d^4 p}{(2\pi)^4} [iS_{Nii}^{>}(p) i\Delta_\phi^{<}(p-k) iS_{\ell bb}^{<}(k) - iS_{Nii}^{<}(p) i\Delta_\phi^{>}(p-k) iS_{\ell bb}^{>}(k)] \\ &\quad + \mathcal{C}_{\ell ab}^{CPV}, \end{aligned} \quad (7)$$

where we take $i\Delta_\phi^{<,>}$ to be of equilibrium form. Note that while the off-diagonal correlations within ℓ and N , which are always out-of-equilibrium, can be neglected for calculating \mathcal{C}_ℓ^Y to second order in Y , within the CP violating source \mathcal{C}_ℓ^{CPV} , these off-diagonal correlations are essential for the asymmetry that we calculate in Eq. (27) below.

For definiteness, we assume strong washout, *i.e.* $M_{Ni} \gg T$ at the time of Leptogenesis. (Due to the exponential Maxwell-suppression, masses of a factor of a few above the temperature are enough for sufficiently accurate approximations.) This allows us to neglect quantum statistical blocking and enhancement factors on many occasions and to use Maxwell distributions instead of Fermi-Dirac or Bose-Einstein distributions. We parametrise the deviation of the neutrino distribution from equilibrium by a pseudo-chemical potential μ_{Ni} as

$$\delta f_{Ni}(p) = \frac{\mu_{Ni}}{T} e^{-\beta \sqrt{\mathbf{p}^2 + M_{Ni}^2}}. \quad (8)$$

While in the present setup, there are no interactions that directly force kinetic equilibrium of the sterile neutrinos, it turns out that above parametrisation indeed gives a good approximation to the distribution function of sterile neutrinos in strong washout scenarios [26, 41].

We now aim to compute the off-diagonal distributions $f_{\ell ab}$ and $\bar{f}_{\ell ab}$, $a \neq b$. In kinetic equilibrium, these can be inferred from the number densities of leptons and anti-leptons,

$$\delta n_{\ell ab}^+ = -2 \int_0^\infty \frac{dk^0}{2\pi} \int \frac{d^3k}{(2\pi)^3} g_{-ab}^{<, >}(k), \quad (9a)$$

$$\delta n_{\ell ab}^- = -2 \int_{-\infty}^0 \frac{dk^0}{2\pi} \int \frac{d^3k}{(2\pi)^3} g_{+ab}^{<, >}(k). \quad (9b)$$

(Note that \pm on $\delta n_{\ell ab}^\pm$ refers to the particle and anti-particle, whereas on $g_{\pm ab}^{<, >}$, it refers to helicity.) The prefix δ indicates that these quantities are deviations of the number densities from their equilibrium values. Consequently, the charge density $q_{\ell ab} = \delta n_{\ell ab}^+ - \delta n_{\ell ab}^-$ can deviate from zero. Pair creation and annihilation processes tend to strongly suppress the combination $\delta n_{\ell ab}^+ + \delta n_{\ell ab}^-$. However, it turns out that for small Yukawa couplings, the finite value of this combination limits the maximal resonant enhancement of the asymmetry, *cf.* Eq. (17) below. For this reason, equations for $\delta n_{\ell ab}^\pm$ rather than just $q_{\ell ab}$ need to be derived, as a generalisation of the methods introduced in Ref. [14].

In order to obtain these densities when the time-derivative in the kinetic equation (2) is small compared to the effective mass difference $\zeta_{aa}^\# - \zeta_{bb}^\#$, it is useful to multiply these equations by $|\mathbf{k}|$ and then to integrate over d^4k . Doing so, we encounter on the right-hand side

$$-\int_0^\infty \frac{dk^0}{2\pi} \int \frac{d^3k}{(2\pi)^3} |\mathbf{k}| \text{tr} \mathcal{C}_\ell^Y(k) = \int_{-\infty}^0 \frac{dk^0}{2\pi} \int \frac{d^3k}{(2\pi)^3} |\mathbf{k}| \text{tr} \mathcal{C}_\ell^Y(k) =: -\sum_i Y_{ai}^\dagger Y_{ib} B_i^Y. \quad (10)$$

In the limit of strong washout, where the quantum statistical distributions can be approximated by Maxwell distributions, we obtain

$$B_i^Y \approx -\frac{T^{\frac{3}{2}} M_{Ni}^{\frac{7}{2}} \mu_{Ni}}{2^{\frac{13}{2}} \pi^{\frac{5}{2}} T} e^{-\frac{M_{Ni}}{T}}. \quad (11)$$

Clearly, this term is exponentially suppressed for large M_{Ni} . It can be interpreted as the CP -violating cut that appears in the resummed lepton propagator of Figure 1(C), and since the reaction $\ell \leftrightarrow \phi^* + N_i$ is kinematically forbidden in the vacuum, this cut is purely thermal.

The interactions mediated by the Yukawa couplings h lead to the decay of off-diagonal correlations of the lepton doublets. The relevant contribution to the collision term is

given by [27]

$$\mathcal{C}_{\ell ab}^h(k) \approx h_{ac}^\dagger h_{cd} \int \frac{d^4 p}{(2\pi)^4} [\mathrm{i} S_{\mathrm{Rcc}}^>(p) \mathrm{i} \Delta_\phi^>(k-p) - \mathrm{i} S_{\mathrm{Rcc}}^<(p) \mathrm{i} \Delta_\phi^<(k-p)] \mathrm{i} \delta S_{\ell cb}(k). \quad (12)$$

In the integrated kinetic equations, then the following terms occur:

$$\begin{aligned} - \int_0^\infty \frac{dk^0}{2\pi} \int \frac{d^3 k}{(2\pi)^3} |\mathbf{k}| \mathrm{tr} \mathcal{C}_{\ell ab}^h(k) &= -h_{ac}^\dagger h_{cd} \int_0^\infty \frac{dk^0}{2\pi} \int \frac{d^3 k}{(2\pi)^3} \int \frac{d^4 p}{(2\pi)^4} |\mathbf{k}| \\ &\times \mathrm{tr} [\mathrm{i} S_{\mathrm{Rcc}}^>(p) \mathrm{i} \Delta_\phi^>(k-p) - \mathrm{i} S_{\mathrm{Rcc}}^<(p) \mathrm{i} \Delta_\phi^<(k-p)] \mathrm{i} \delta S_{\ell cb}(k) \\ &= -h_{ac}^\dagger h_{cd} \delta n_{\ell ab}^+ \int_0^\infty \frac{dk^0}{2\pi} \int \frac{d^3 k}{(2\pi)^3} \int \frac{d^4 p}{(2\pi)^4} |\mathbf{k}| \\ &\times \mathrm{tr} [\mathrm{i} S_{\mathrm{Rcc}}^>(p) \mathrm{i} \Delta_\phi^>(k-p) - \mathrm{i} S_{\mathrm{Rcc}}^<(p) \mathrm{i} \Delta_\phi^<(k-p)] 2S_{\ell cb}^{\mathcal{A}}(k) \frac{12\beta^3 e^{\beta k^0}}{(e^{\beta k^0} + 1)^2} \\ &=: -B_\ell^\sharp [h^\dagger h \delta n_\ell^+]_{ab} \end{aligned} \quad (13)$$

and likewise

$$\int_{-\infty}^0 \frac{dk^0}{2\pi} \int \frac{d^3 k}{(2\pi)^3} |\mathbf{k}| \mathrm{tr} \mathcal{C}_{\ell ab}^h(k) = -B_\ell^\sharp [h^\dagger h \delta n_\ell^-]_{ab}. \quad (14)$$

With these coefficients, the integrated kinetic equations are

$$\frac{54\zeta(3)T}{\pi^2} \partial_\eta \delta n_{\ell ab}^+ + \mathrm{i} \frac{(h_{aa}^2 - h_{bb}^2) T^2}{16} \delta n_{\ell ab}^+ \quad (15a)$$

$$= - \sum_i Y_{ai}^\dagger Y_{ib} B_i^Y - (h_{aa}^2 + h_{bb}^2) B_\ell^\sharp \delta n_{\ell ab}^+ - B_\ell^g (\delta n_{\ell ab}^+ + \delta n_{\ell ab}^-),$$

$$\frac{54\zeta(3)T}{\pi^2} \partial_\eta \delta n_{\ell ab}^- - \mathrm{i} \frac{(h_{aa}^2 - h_{bb}^2) T^2}{16} \delta n_{\ell ab}^- \quad (15b)$$

$$= - \sum_i Y_{ai}^\dagger Y_{ib} B_i^Y - (h_{aa}^2 + h_{bb}^2) B_\ell^\sharp \delta n_{\ell ab}^- - B_\ell^g (\delta n_{\ell ab}^+ + \delta n_{\ell ab}^-).$$

In principle, there also occur terms

$$\propto ([Y^\dagger Y]_{aa} + [Y^\dagger Y]_{bb}) \delta n_{\ell ab}^\pm \quad (16)$$

on the right hand sides of these equations. However, we assume here that the dominant flavour-sensitive interaction is mediated by the lepton-Yukawa couplings. In particular, $h_{\tau\tau}$ dominates over the relevant elements of Y , which is valid provided Leptogenesis occurs at temperatures below 10^{12} GeV.

The coefficient B_ℓ^\sharp is the averaged rate for flavour-sensitive processes that are mediated by the coupling h . As the Higgs boson as well as the Standard Model leptons are massless in the symmetric Electroweak phase, these reactions require the radiation of an additional gauge boson. A systematic calculation is currently on the way, but from the similar problem of the production of light right-handed neutrinos [42–46], we may estimate that $B_\ell^\sharp = 1.0 \times 10^{-2} T^2$, see the Appendix A.

Pair creation and annihilation processes may change $\delta n_{\ell ab}^\pm$. Even though these processes are flavour blind, we will see below that in interplay with flavour-sensitive processes, they can be important for the suppression of flavour off-diagonal correlations. Accounting for the large number of Standard Model degrees of freedom, these processes are dominated by the s -channel exchange of gauge bosons, and we estimate these within Appendix A as $B_\ell^g = 1.7 \times 10^{-3} T^2$.

When neglecting the time-derivative, Eqs. (15) yield the induced off-diagonal ($a \neq b$) correlations

$$q_{\ell ab} \equiv q_{\ell ab}(\eta \rightarrow \infty) \quad (17a)$$

$$\begin{aligned} &= \delta n_{\ell ab}^+ - \delta n_{\ell ab}^- = i \frac{(h_{aa}^2 - h_{bb}^2)(T^2/8) \sum_i Y_{ai}^\dagger Y_{ib} B_i^Y}{[(h_{aa}^2 - h_{bb}^2)T^2/16]^2 + (h_{aa}^2 + h_{bb}^2)B_\ell^\sharp [2B_\ell^g + (h_{aa}^2 + h_{bb}^2)B_\ell^\sharp]} \\ &=: (\mathcal{Q}_{\ell ab}/T^2) \sum_i Y_{ai}^\dagger Y_{ib} B_i^Y, \\ \delta n_{\ell ab}^+ + \delta n_{\ell ab}^- &= - \frac{2(h_{aa}^2 + h_{bb}^2)B_\ell^\sharp \sum_i Y_{ai}^\dagger Y_{ib} B_i^Y}{[(h_{aa}^2 - h_{bb}^2)T^2/16]^2 + (h_{aa}^2 + h_{bb}^2)B_\ell^\sharp [2B_\ell^g + (h_{aa}^2 + h_{bb}^2)B_\ell^\sharp]}. \end{aligned} \quad (17b)$$

The general solution to Eqs. (15) involves the damping rates

$$\Gamma_{q_{\ell ab}}^\pm = \frac{\pi^2}{54\zeta(3)T} \left(B_\ell^g + (h_{aa}^2 + h_{bb}^2)B_\ell^\sharp \pm \sqrt{B_\ell^{g2} - [(h_{aa}^2 - h_{bb}^2)T^2/16]^2} \right), \quad (18)$$

and is given by

$$q_{\ell ab}(\eta) = q_{\ell ab}(\eta \rightarrow \infty) + Q_1 e^{-\Gamma_{q_{\ell ab}}^+ \eta} + Q_2 e^{-\Gamma_{q_{\ell ab}}^- \eta}. \quad (19)$$

This implies that it takes a finite time for these CP -even off-diagonal correlations to build up. In the present case, the rate for this is given by $\Gamma_{q_{\ell\tau\tau\perp}}^- \sim h_{\tau\tau}^2 T$. This in particular implies that in the regime of flavoured Leptogenesis below 10^{12} GeV, when the interactions mediated by $h_{\tau\tau}$ are in equilibrium, flavour off-diagonal lepton correlations are present, and their magnitude is given by Eqs. (17).

We express the resonant enhancement of the asymmetries through the factor

$$\mathcal{Q}_{\ell ab} = \frac{(h_{aa}^2 - h_{bb}^2)(T^4/8)}{[(h_{aa}^2 - h_{bb}^2)/16]^2 T^4 + (h_{aa}^2 + h_{bb}^2)B_\ell^\sharp [2B_\ell^g + (h_{aa}^2 + h_{bb}^2)B_\ell^\sharp]}. \quad (20)$$

The fact that $\delta n_{\ell ab}^+ + \delta n_{\ell ab}^-$ is suppressed but not vanishing implies that kinetic equilibrium mediated by pair creation and annihilation processes is only approximately established. Processes of kinetic equilibration that do not mediate between δn_{ℓ}^+ and δn_{ℓ}^- , *i.e.* which do not correspond to pair creation and annihilation, are however not conflicting with the solution (17). We therefore describe the distribution functions in terms of the number densities as

$$\delta f_{\ell}(\mathbf{k}) = \frac{12\delta n_{\ell}^+}{T^3} \frac{e^{|\mathbf{k}|/T}}{(e^{|\mathbf{k}|/T} + 1)^2}, \quad (21a)$$

$$\delta \bar{f}_{\ell}(\mathbf{k}) = \frac{12\delta n_{\ell}^-}{T^3} \frac{e^{|\mathbf{k}|/T}}{(e^{|\mathbf{k}|/T} + 1)^2}. \quad (21b)$$

Notice also that from Eqs. (17) it follows that $n_{\ell ab}^+ = n_{\ell ab}^{-*}$, $n_{\ell ab}^{\pm} = n_{\ell ba}^{\pm*}$ and therefore $n_{\ell ab}^+ = n_{\ell ba}^-$. This implies that the flavour off-diagonal lepton correlations are CP -even,

$$P_{\text{Li}} S_{\ell ab}^{fg}(p) = C \left[P_{\text{Li}} S_{\ell ba}^{gf}(-p) \right]^t C^{\dagger} + \mathcal{O}(Y^4), \quad (22)$$

such that to this end, the lepton asymmetry is not yet broken.

3 Asymmetries of Lepton Doublets and Singlet Neutrinos

In this Section, we calculate the CP -odd charge asymmetries within the diagonal components of the distribution functions of lepton doublets as well as the helicity asymmetries within the singlet neutrinos, that are induced by the off-diagonal, CP -even correlations of lepton doublets. In the strong washout scenario, helicity asymmetries within the right-handed neutrinos are quantitatively irrelevant, because of the Majorana masses, which effectively erase such asymmetries when the right-handed neutrinos decay. However, helicity asymmetries can be of crucial importance in weak washout scenarios [10, 32–35].

In the present context, we are interested in the helicity asymmetries, because the lepton-doublet propagator and its self-energy correction do not encompass lepton-number violating combinations of operators, such that the total source for the lepton asymmetry plus the helicity asymmetry in the right-handed neutrinos should vanish. This constraint provides a useful consistency check that we perform in the present Section. In spite of the vanishing of the total source, the asymmetry of the singlet neutrinos is completely washed out in the decays, that proceed via the Majorana mass term within the strong washout regime. In contrast, the active leptons experience finite washout rates from the inverse decays, such that a total lepton asymmetry remains at freeze out.

The self-energy for the right-handed neutrino is [8, 10, 26]

$$\begin{aligned} i\Sigma_{Nij}^{fg}(k) = & g_w Y_{ia} Y_{bj}^\dagger \int \frac{d^4 p}{(2\pi)^4} P_L iS_{\ell ab}^{fg}(p) i\Delta_\phi^{fg}(k-p) \\ & + g_w Y_{ia}^* Y_{bj}^t \int \frac{d^4 p}{(2\pi)^4} C \left[P_L iS_{\ell ba}^{gf}(-p) \right]^t C^\dagger i\Delta_\phi^{gf}(p-k), \end{aligned} \quad (23)$$

where $g_w = 2$ is the dimension of the $SU(2)_L$ representation of the lepton doublet. The self-energy enters into the collision term for the right-handed neutrino as

$$\text{tr} \mathcal{C}_N(k) = \text{tr} \left[i\Sigma_N^>(k) iS_N^<(k) - i\Sigma_N^<(k) iS_N^>(k) \right]. \quad (24)$$

A CP - and helicity-violating contribution \mathcal{C}_N^{CPV} is induced by the flavour off-diagonal correlations of the lepton doublets. We denote this as the source term

$$\begin{aligned} \mathcal{S}_{Nij} = & \int \frac{d^4 k}{(2\pi)^4} \frac{1}{4} \text{tr} \left[\mathcal{C}_N^{CPV}(k) + \mathcal{C}_N^{CPV\dagger}(k) \right]_{ij} \\ = & \frac{1}{2} \sum_{\substack{a,b \\ a \neq b}} \int \frac{d^4 k}{(2\pi)^4} \int \frac{d^4 p}{(2\pi)^4} g_w Y_{ia} Y_{bj}^\dagger \text{tr} \left[i\delta S_{\ell ab}(p) (i\Delta_\phi^>(k-p) [iS_{Ni}^<(k) + iS_{Njj}^<(k)] \right. \\ & \left. - i\Delta_\phi^<(k-p) [iS_{Ni}^>(k) + iS_{Njj}^>(k)] \right]. \end{aligned} \quad (25)$$

We have made use here of the $\mathcal{O}(Y^2)$ lepton number conservation, Eq. (22), and assumed that there is no charge asymmetry within the Higgs field, such that $i\Delta_\phi^{<,>}(p) = i\Delta_\phi^{>,<}(-p)$. Moreover, the off-diagonal correlations within the right-handed neutrinos, which are important for the usually considered CP -violating sources for Leptogenesis, are neglected here. (An example with a quantitative comparison of the source from mixing and decays of right-handed neutrinos and the source from mixing lepton doublets is presented in Section 4.)

The source term can be expressed to enter the integrated kinetic equation for the right-handed neutrinos in the form

$$\frac{1}{2} \partial_\eta \int \frac{d^4 k}{(2\pi)^4} i \text{tr} [\gamma^0 S_{Nij}^{<,>}(k)] = -\mathcal{S}_{Nij}. \quad (26)$$

The integral on the left hand side corresponds to the rate of producing a helicity asymmetry, which is in the limit of vanishing Majorana masses the same as an asymmetry between right-handed neutrinos and anti-neutrinos. Note that this integrated kinetic equation does not account for the total production of singlet neutrinos. In order to obtain these rates, the collision term is to be integrated from 0 to ∞ rather than from $-\infty$ to ∞ over dk^0 [26]. The factor $\frac{1}{2}$ on the right hand side is a suitable normalisation, because in writing the Majorana field as a four component spinor, the number of degrees of freedom is doubled, though they are related by the Majorana constraint.

As for the lepton collision term \mathcal{C}_ℓ^Y , the approximation (7) is used in order to calculate the CP -even off-diagonal correlations in terms of the deviation of the right-handed neutrinos from equilibrium. For that purpose, we have taken S_ℓ to be of equilibrium form under the integral. Now, we want to obtain the CP -odd asymmetry in the lepton doublets that is induced by these off-diagonal correlations. It follows from integrating \mathcal{C}_ℓ^{CPV} as

$$\begin{aligned} \mathcal{S}_{\ell ab} &= \int \frac{d^4 k}{(2\pi)^4} \frac{1}{2} \text{tr} \left[\mathcal{C}_\ell^{CPV}(k) + \mathcal{C}_\ell^{CPV\dagger}(k) \right]_{ab} \\ &= \frac{1}{2} \sum_{\substack{i,c \\ c \neq b}} \int \frac{d^4 k}{(2\pi)^4} \int \frac{d^4 p}{(2\pi)^4} \text{tr} \left[(P_R i S_{Nii}^>(p) i \Delta_\phi^<(p-k) - P_R i S_{Nii}^<(p) i \Delta_\phi^>(p-k)) \right. \\ &\quad \left. \times (Y_{ai}^\dagger Y_{ic} i \delta S_{\ell cb}(k) + i \delta S_{\ell ac}(k) Y_{ci}^\dagger Y_{ib}) \right]. \end{aligned} \quad (27)$$

Besides this source, at fourth order in Y , there are also the standard contributions from direct CP -violation in the decays and inverse decays of right-handed neutrinos as well as from their mixing [2]. These add linearly to the asymmetry from lepton-doublet mixing, that is the main topic of the present work. For simplicity, we do not account for the standard sources of asymmetry in most of the following discussion and refer to Refs. [8, 10, 13, 26, 27] for the calculation of these contributions within the CTP formalism. In Section 4 however, we present a comparison of the asymmetry resulting from lepton mixing to the asymmetry that results from the standard sources.

When expressing the kinetic equation for the lepton doublets as

$$\partial_\eta \int \frac{d^4 k}{(2\pi)^4} i \text{tr} [\gamma^0 S_{Nij}^{<,>}(k)] = - \int \frac{d^4 k}{(2\pi)^4} \frac{1}{2} \text{tr} [\mathcal{C}_\ell(k) + \mathcal{C}_\ell^\dagger(k)]_{ab} \quad (28)$$

and observing that

$$\text{tr}[\mathcal{S}_N] = -g_w \text{tr}[\mathcal{S}_\ell], \quad (29)$$

we see that the sum of the helicity asymmetry within the right-handed neutrinos and the charge asymmetry in the lepton-doublets is conserved, unless taking proper account of the helicity-flipping Majorana masses M_N . This is expected, since \mathcal{Z}_ℓ , the self-energy that leads to the mixing of the lepton doublets, does not encompass lepton-number violating operators in the form of odd powers of the Majorana masses M_N within the approximation represented by the diagram in Figure 1(c) (*i.e.* when accounting for lepton-doublet mixing only and not for the mixing and the direct decays of the right-handed neutrinos).

Now, since in the strong washout scenario, $M_{Nii} \gg T$, the right-handed neutrinos are non-relativistic, and they decay at tree-level with equal likelihood into leptons and anti-leptons. This is different for relativistic right-handed neutrinos, that are converted to leptons via scattering processes with the particles in the plasma. These reactions are

approximately helicity conserving. In the following, we calculate the asymmetry in the strong washout scenario, and for this reason, we only have to account for the production and the washout of the asymmetry within the leptons ℓ , because the helicity asymmetry within the right-handed neutrinos is rapidly violated by the Majorana masses. In the strong washout regime, all energies of particles that participate in decay and inverse decay processes are much larger than the temperature. One may therefore approximate Eqs. (21) by their Maxwell forms

$$\delta f_\ell(\mathbf{k}) = \frac{12\delta n_\ell^+}{T^3} e^{-|\mathbf{k}|/T}, \quad (30a)$$

$$\delta \bar{f}_\ell(\mathbf{k}) = \frac{12\delta n_\ell^-}{T^3} e^{-|\mathbf{k}|/T}. \quad (30b)$$

We then substitute above approximations into Eqs. (17), and these into the source term for the lepton asymmetry (27). It enters the kinetic equations (2), when integrating these over dk^0 ,

$$\begin{aligned} \partial_\eta q_{\ell aa} &= \sum_{ic} \frac{1}{32\pi^3} \left[Y_{ai}^\dagger Y_{ic} \frac{6q_{\ell ca}}{T^3} + \frac{6q_{\ell ac}}{T^3} Y_{ci}^\dagger Y_{ib} \right] M_{Ni}^3 T K_1(M_{Ni}/T) + \int \frac{d^4 k}{(2\pi)^4} C_{\ell aa}^Y(k) \quad (31) \\ &= \sum_{\substack{ijc \\ c \neq a}} \frac{3}{16\pi^3} \frac{M_{Ni}^3}{T^4} K_1(M_{Ni}/T) B_j^Y \mathcal{Q}_{\ell cai} \left(Y_{ai}^\dagger Y_{ic} Y_{cj}^\dagger Y_{ja} - Y_{aj}^\dagger Y_{jc} Y_{ci}^\dagger Y_{ia} \right) \\ &\quad + \int \frac{d^4 k}{(2\pi)^4} C_{\ell aa}^Y(k). \end{aligned}$$

Here, we have evaluated the source term (27) explicitly in the limit of strong washout, while the remaining integral corresponds to the washout term.

4 Freeze-Out Asymmetries in the Strong Washout Regime

We now calculate the freeze-out value of the asymmetry produced in the out-of-equilibrium decays of the individual N_i in the strong washout regime. The total asymmetry is then obtained as the sum of the particular asymmetries. As usual, it is convenient to parametrise the evolution by

$$z_i = M_{Ni}/T. \quad (32)$$

The scale factor a can be expressed through the comoving temperature a_R as

$$a = a_R \eta = a_R \frac{z_i}{M_{Ni}}, \quad a_R = \frac{m_{\text{Pl}}}{2} \sqrt{\frac{45}{\pi^3 g_\star}}, \quad (33)$$

where $g_\star = 106.75$ is the number of relativistic degrees of freedom of the Standard Model at high temperatures. The physical temperature is given by $T = a_R/a = M_{Ni}/z_i$. In order to take account of the expansion of the Universe, in all kinetic equations written down to this end, the mass terms should be multiplied by the scale factor a and all temperatures be replaced by the comoving temperature a_R , *cf.* Ref [26]. The kinetic equations then take the form

$$\frac{dY_{\ell a}^{Ni}}{dz_i} = \bar{S}_{\ell aa}^{Ni}(Y_{Ni} - Y_{Ni}^{\text{eq}}) + \bar{W}_{\ell a} Y_{\ell a}, \quad (34a)$$

$$\frac{dY_{Nk}}{dz_i} = \bar{C}_{Nk}(Y_{Nk} - Y_{Nk}^{\text{eq}}), \quad (34b)$$

where $Y_{\ell a} = q_{\ell aa}/s$ and $Y_{Ni} = n_{Ni}/s$, and s is the entropy density. The various distributions, collision and source terms are (*cf.* Ref. [14])

$$\bar{S}_{\ell aa}^{Ni} = - \sum_{\substack{j,c \\ j \neq i, c \neq a}} \frac{3a_R z_i^{\frac{9}{2}} e^{-\frac{M_{Nj}}{M_{Ni}} z_i} M_{Nj}^{\frac{7}{2}} [YY^\dagger]_{ii}}{2^{23/2} \pi^{7/2} M_{Ni}^{\frac{9}{2}} [YY^\dagger]_{jj}} \mathcal{Q}_{\ell ac i} \left(Y_{ai}^\dagger Y_{ic} Y_{cj}^\dagger Y_{ja} - Y_{aj}^\dagger Y_{jc} Y_{ci}^\dagger Y_{ia} \right), \quad (35a)$$

$$Y_{Nk}^{\text{eq}} = 2^{-1/2} \pi^{-3/2} \left(\frac{M_{Nk}}{M_{Ni}} \right)^{3/2} z_i^{3/2} a_R^3 e^{-z_i M_{Nk}/M_{Ni}} / s, \quad (35b)$$

$$\bar{C}_{Nk} = \frac{g_w}{16\pi} \sum_a Y_{ka} Y_{ak}^\dagger a_R z_i \frac{M_{Nk}}{M_{Ni}^2}, \quad (35c)$$

$$\bar{W}_{\ell a}^{Ni} = - \sum_k Y_{ak}^\dagger Y_{ka} \frac{3z_i^{5/2}}{2^{9/2} \pi^{5/2}} \left(\frac{M_{Nk}}{M_{Ni}} \right)^{5/2} \frac{a_R}{M_{Ni}} e^{-z_i M_{Nk}/M_{Ni}} =: \sum_k B_{\ell ak}^{Ni} z_i^{5/2} e^{-z_i M_{Nk}/M_{Ni}}. \quad (35d)$$

To the leading order in deviations from equilibrium, $Y_{Nj} - Y_{Nj}^{\text{eq}}$, the formal solution to Eqs. (34) is given by [47, 48]

$$\begin{aligned} Y_{\ell a}^{Ni}(z_i) &= \int_0^{z_i} dz' S_{\ell aa}^{Ni} \frac{d}{dz'} \frac{Y_{Ni}^{\text{eq}}}{\bar{C}_{Ni}} e^{\int_{z'}^{z_i} dz'' \bar{W}_{\ell a}^{Ni}(z'')} \\ &= \int_0^{z_i} dz' a_a^{Ni}(z') e^{-\int_{z'}^{z_i} dz'' \sum_k B_{\ell ak} z''^{5/2} \exp(-z'' M_{Nk}/M_{Ni})}, \end{aligned} \quad (36)$$

where we define

$$\begin{aligned} a_a^{Ni}(z_i) &= S_{\ell aa}^{Ni} \frac{d}{dz'} \frac{Y_{Ni}^{\text{eq}}}{\bar{C}_{Ni}} = \sum_{\substack{j,c \\ j \neq i, c \neq a}} \frac{3a_R^3 z_i^5 e^{-\frac{M_{Ni}+M_{Nj}}{M_{Ni}} z_i} \left(\frac{M_{Nj}}{M_{Ni}} \right)^{\frac{7}{2}}}{2^8 \pi^4 s [YY^\dagger]_{jj}} \\ &\quad \times \mathcal{Q}_{\ell ac i} \left(Y_{ai}^\dagger Y_{ic} Y_{cj}^\dagger Y_{ja} - Y_{aj}^\dagger Y_{jc} Y_{ci}^\dagger Y_{ia} \right). \end{aligned} \quad (37)$$

For simplicity, we now consider Leptogenesis from two right-handed neutrinos $N_{i,j}$. This is relevant in the situation when there are only two right-handed neutrinos in the theory, when additional right-handed neutrinos are much heavier than N_1 and N_2 or when these decouple due to the smallness of their Yukawa couplings.

The exponent in Eq. (36) is then extremal for $z' = z_f$ with

$$\sum_k B_{lak} z_f^{5/2} e^{-z_f M_{Nk}/M_{Ni}} = \frac{M_{Ni} + M_{Nj}}{M_{Ni}}, \quad (38)$$

which we solve numerically for z_f , what determines the freeze-out temperature. Laplace's steepest descent method then yields the flavoured freeze-out asymmetries (*cf.* Refs. [47, 48])

$$Y_{\ell a}^{Ni}(z = \infty) = a_{1a}(z_f) \sqrt{\frac{2\pi}{\sum_k B_{lak} z_f^{5/2} e^{-z_f M_{Nk}/M_{Ni}}}} e^{-\int_{z_f}^{\infty} \sum_k dz'' B_{lak} z''^{5/2} \exp(-z'' M_{Nk}/M_{Ni})}. \quad (39)$$

Finally, the total lepton asymmetry is the sum

$$Y_\ell = \sum_{i,a} Y_{\ell a}^{Ni}. \quad (40)$$

In order to assess the relative importance of the asymmetry from mixing lepton doublets, we compare it to the lepton asymmetry that results from the wave function and vertex corrections to the decays of right-handed neutrinos N . We can simply express

$$\frac{dY_{\ell a}^{Ni}}{dz} = \varepsilon_a^{Ni} \bar{C}_{Ni} (Y_{Ni} - Y_{Ni}^{\text{eq}}) + \bar{W}_{\ell a}^{Ni} Y_{\ell a}^{Ni}, \quad (41)$$

where again, Y_ℓ^{Ni} is the asymmetry that results from the decay of N_i . The decay asymmetry is [2, 49]

$$\varepsilon_a^{Ni} = \frac{3}{16\pi [YY^\dagger]_{ii}} \sum_{\substack{j,b \\ j \neq i}} \left\{ \text{Im} \left[Y_{ai}^\dagger Y_{ib}^* Y_{bj}^t Y_{ja} \right] \frac{\xi(x_j)}{\sqrt{x_j}} + \text{Im} \left[Y_{ai}^\dagger Y_{ib} Y_{bj}^\dagger Y_{ja} \right] \frac{2}{3(x_j - 1)} \right\}, \quad (42)$$

where $x_j = M_{Nj}/M_{Ni}$ and

$$\xi(x) = \frac{2}{3} x \left[(1+x) \log \frac{1+x}{x} - \frac{2-x}{1-x} \right]. \quad (43)$$

The freeze-out temperature is now determined by

$$\sum_k B_{lak}^{Ni} z_f^{5/2} e^{-z_f M_{Nk}/M_{Ni}} = 1, \quad (44)$$

where $B_{\ell a k}^{Ni}$ is given by Eq. (35d). The factor that appears in the formal solution (36) for $Y_{\ell a}^{Ni}$ is now

$$a_a^{Ni}(z_i) = -\varepsilon_a^{Ni} \frac{a_R^3}{s} 2^{-\frac{1}{2}} \pi^{-\frac{3}{2}} z_i^{\frac{3}{2}} e^{-z_i}. \quad (45)$$

We now pick some particular, but illustrative points in parameter space in order to compare the asymmetry from the mixing of lepton doublets to the asymmetry from the wave function and vertex corrections to the decay parameter of right-handed neutrinos. Parameters consistent with neutrino oscillations can be constructed with the help of the relation [50]

$$Y^\dagger = V^{\Delta\dagger} U_\nu \sqrt{m_\nu^{\text{diag}}} \mathcal{R} \sqrt{M_N} \frac{\sqrt{2}}{v}, \quad (46)$$

where $m_\nu^{\text{diag}} = \text{diag}(m_1, m_2, m_3)$ is the diagonal mass matrix of active leptons, $M_N = \text{diag}(M_{N11}, M_{N22}, M_{N33})$ and $v = 246 \text{ GeV}$. The PMNS matrix U_ν is

$$U_\nu = V^{(23)} U_\delta V^{(13)} U_{-\delta} V^{(12)} \text{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1) \quad (47)$$

with $U_{\pm\delta} = \text{diag}(e^{\mp i\delta/2}, 1, e^{\pm i\delta/2})$. The non-zero entries of the matrices $V^{(ij)}$ are

$$V_{ii}^{(ij)} = V_{jj}^{(ij)} = \cos \theta_{ij}, \quad V_{ij}^{(ij)} = \sin \theta_{ij}, \quad V_{ji}^{(ij)} = -\sin \theta_{ij}, \quad V_{kk}^{(ij)} = 1 \quad (k \neq i, j). \quad (48)$$

The θ_{ij} are the mixing angles of the active neutrinos, and α_1, α_2 and δ are CP -violating phases. The matrix \mathcal{R} must satisfy $\mathcal{R}^t \mathcal{R} = \mathbb{1}$. We parametrise it by the complex angles ω_{ij} and the matrices $\mathcal{R}^{(ij)}$ with non-zero entries

$$\mathcal{R}_{ii}^{(ij)} = \mathcal{R}_{jj}^{(ij)} = \cos \omega_{ij}, \quad \mathcal{R}_{ij}^{(ij)} = \sin \omega_{ij}, \quad \mathcal{R}_{ji}^{(ij)} = -\sin \omega_{ij}, \quad \mathcal{R}_{kk}^{(ij)} = 1 \quad (k \neq i, l), \quad (49)$$

$$\mathcal{R} = \mathcal{R}^{(23)} \mathcal{R}^{(13)} \mathcal{R}^{(12)}.$$

We take $M_{N3} \gg M_{N1,2}$ or alternatively, make the assumption that there is no right-handed neutrino N_3 or simply that the Yukawa couplings of N_3 are negligibly small. The parameter space can then be restricted by the choice $\omega_{13} = \pi/2$ and $\omega_{23} = 0$. Additional parameters that we choose are listed in Table 1. Finally, V^Δ is a unitary matrix that brings Y to a form, such that only asymmetries in two flavours τ and σ are produced, where σ is a linear combination of e and μ , see Ref. [27] for the explicit construction. (Explicitly, this means that $Y_{2\sigma\perp} = Y_{3\sigma\perp} = Y_{3\sigma} = Y_{3\tau} = 0$, while the remaining entries are generically non-zero, and σ_\perp is the linear flavour combination orthogonal to τ and σ .)

The one remaining parameter is M_{N2} , that we vary for values larger than M_{N1} . In Figure 2, we compare the resulting asymmetry from lepton-doublet mixing to the asymmetry from mixing and direct decays of right-handed neutrinos, where we take account of the decays of both, N_1 and N_2 . For $M_{N2} \rightarrow M_{N1}$ (degenerate regime), the asymmetry from the mixing of right-handed neutrinos is strongly enhanced, which is known as

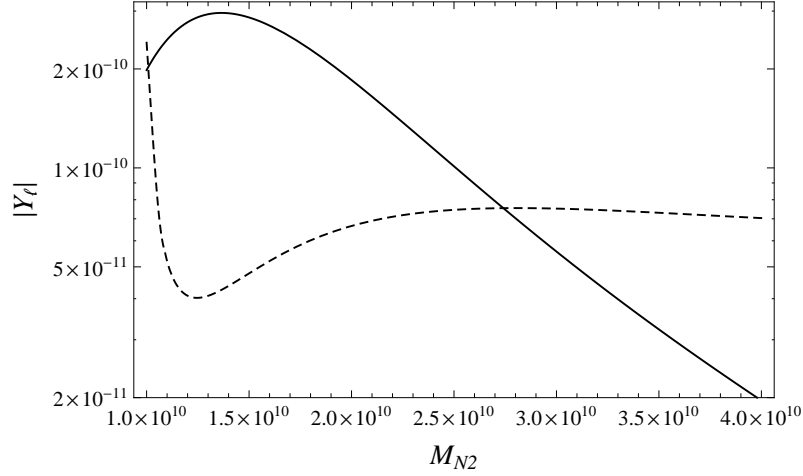


Figure 2: Lepton asymmetry from mixing leptons (solid line) and from wave-function and vertex corrections for the right-handed neutrinos (dashed line). The model parameters are listed in Table 1.

| | | | |
|--------------------|---------|---------------|----------------|
| m_1 | 0 | α_1 | 0 |
| m_2 | 8.7 meV | α_2 | $\pi/2$ |
| $\sin \theta_{12}$ | 0.55 | ω_{12} | $-\pi/2 + i/2$ |
| $\sin \theta_{23}$ | 0.63 | ω_{13} | $\pi/2$ |
| $\sin \theta_{13}$ | 0.16 | ω_{23} | 0 |
| δ | 0 | M_{N1} | 10^{10} GeV |

Table 1: Parameters for the numerical example in Figure 2.

resonant Leptogenesis. The asymmetry from lepton-doublet mixing also is strongest for $M_{N2} \sim M_{N1}$. For larger values of M_{N2} , we clearly see the exponential suppression of the asymmetry, because the kinematic cuts that give rise to the CP -violation are purely thermal and are Maxwell-suppressed for large values of M_{N2} (hierarchical regime), *cf.* Eq. (11). For intermediate values of M_{N2} , between the degenerate and the hierarchical regimes, the asymmetry from mixing lepton doublets dominates over the usually considered asymmetry from mixing and direct decays of right-handed neutrinos.

This example should illustrate that for $M_{N1} \sim M_{N2}$ (more precisely, when the right-handed neutrino masses are within a factor of a few), there generically is a sizeable contribution to the lepton asymmetry, that originates from lepton mixing. While we have considered here a particular slice of the parameter space, it would be interesting to perform a more systematic parametric study of Leptogenesis from mixing lepton doublets in the future.

5 Maximal Resonant Enhancement in Leptogenesis from Multiple Higgs Doublets

In Ref. [14], Leptogenesis from mixing Higgs doublets is introduced. The model is based on the Lagrangian

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \bar{\psi}_N (i \not{\partial} - M_N) \psi_N + \bar{\psi}_\ell i \not{\partial} \psi_\ell + \sum_k (\partial_\mu \phi_k) (\partial^\mu \phi_k) \\ & - \sum_{kl} M_{\phi kl}^2 \phi_k^* \phi_l - \sum_{mk} (Y_{mk} \bar{\psi}_N \phi_k P_L \psi_{\ell m} + \text{h.c.}) . \end{aligned} \quad (50)$$

Notice that the Yukawa couplings Y here are different from those in the Lagrangian (1). Using the same notations and conventions as in Ref. [14], we obtain the off-diagonal correlations in the number densities of Higgs and anti-Higgs particles as

$$\delta n_{\phi 12}^\pm = 2 \int_0^{\infty, -\infty} \frac{dk^0}{2\pi} \int \frac{d^3 k}{2\pi} k^0 i D_{\phi 12}(k) , \quad (51)$$

where D_ϕ is the Wightman function for the Higgs field.

The kinetic equations for the off-diagonal correlations of the Higgs particles are then (we choose a basis where M_ϕ , that may include thermal corrections, is diagonal)

$$\pm i (M_{\phi 11}^2 - M_{\phi 22}^2) \delta n_{\phi 12}^\pm = -[Y^\dagger Y]_{12} B_\phi^Y - B_\phi^\# \sum_j (y_{j1}^2 + y_{j2}^2) \delta n_{\phi 12}^\pm - B_\phi^g (\delta n_{\phi 12}^+ + \delta n_{\phi 12}^-) . \quad (52)$$

We have assumed here that the flavour-sensitive interactions are dominated by couplings y_{ik} of the Higgs-fields ϕ_k to Standard Model fermions. Note however that the couplings Y and the quartic interactions among the Higgs fields are flavour sensitive as well and should be taken into account when the y_{ik} are not dominating. An estimate for the weighted annihilation rate $B_\phi^g \approx 1.0 \times 10^{-4} T^2$ is given by Eq. (62) in Appendix A. As for $B_\phi^\#$, we do not provide an estimate here, since it is model dependent. If one of the Higgs fields ϕ is the Standard Model Higgs field, this rate will be dominated by the top-quark Yukawa couplings y_{ti} . Because the scattering processes then also involve gluons, we may expect this rate to be enhanced by a factor of a few when compared to $B_\ell^\#$.

Eq. (52) has the solution

$$q_{\phi 12} = \delta n_{\phi 12}^+ - \delta n_{\phi 12}^- = \mathcal{R}_\phi 2i[Y^\dagger Y]_{12} B_\phi^Y , \quad (53a)$$

$$\mathcal{R}_\phi = \frac{M_{\phi 11}^2 - M_{\phi 22}^2}{(M_{\phi 11}^2 - M_{\phi 22}^2)^2 + B_\phi^\# \sum_j (y_{j1}^2 + y_{j2}^2) [B_\phi^\# \sum_j (y_{j1}^2 + y_{j2}^2) + 2B_\phi^g]} , \quad (53b)$$

$$B_\phi^Y = \frac{\mu_N M_N^{7/2} T^{1/2}}{32\sqrt{2}\pi^{5/2}} e^{-M_N/T} . \quad (53c)$$

The enhancement factor \mathcal{R}_ϕ is analogous to the quantity (20) \mathcal{Q}_{lab} for mixing leptons, but we have choose a different dimensionality for convenience and the sake of comparison with Ref. [14].

Following Ref. [14], we then obtain for the freeze-out asymmetry

$$Y_{\ell i}(z = \infty) = \frac{\text{Im}[Y_{i1}Y_{j1}^*Y_{j2}Y_{i2}^*]}{\text{tr}[YY^\dagger]} \frac{135}{64\pi^6 g_\star} M_N^2 \mathcal{R}_\phi \sqrt{\frac{e^{z_{fi}}}{\sqrt{z_{fi}} B_i}} e^{-2z_{fi} - \int_{z_{fi}}^{\infty} dz B_i z^{5/2} e^{-z}}, \quad (54a)$$

$$B_i = [Y^\dagger Y]_{ii} \frac{9}{32\pi^4} \sqrt{\frac{5}{2g_\star}} \frac{m_{Pl}}{M_N}, \quad (54b)$$

$$z_{fi} = -\frac{5}{2} W_{-1} \left((-2/5) \times (2/B_i)^{2/5} \right). \quad (54c)$$

(Note a missing factor $1/\sqrt{B_i}$ in the expression for $Y_{\ell i}$ in Ref. [14].)

6 Summary and Discussion

We have calculated the mixing of Standard Model lepton doublets in the early Universe and studied its impact on Baryogenesis via Leptogenesis. The mixing is driven by non-equilibrium right-handed neutrinos, as described in Section 2. With no deviation from equilibrium, there is no mixing of the lepton doublets. The amount of mixing is quantified by the the correlations of the off-diagonal number densities (17).

Substituting these off-diagonal flavour correlations back into the collision term for the leptons, we obtain the source for the lepton asymmetry, Eqs. (27,31) in Section 3. This asymmetry is opposite to the source for the helicity asymmetry in the right-handed neutrinos (which is however rapidly violated through the Majorana masses).

By calculating the freeze-out value of the lepton asymmetry, we compare the source from lepton mixing to the standard source from mixing and direct decays of right-handed neutrinos in Section 4. The standard result is enhanced for mass-degenerate right-handed neutrinos (resonant Leptogenesis), while the asymmetry from lepton mixing is Maxwell-suppressed for larger mass ratios. In between these regimes, we find that lepton-doublet mixing may be the dominant source of CP -asymmetry for Leptogenesis.

Results for the impact of gauge interactions on the mixing of Higgs-doublets and implications for the freeze-out value of the induced asymmetries are presented in Section 5.

In the future, it would be interesting to perform a more systematic scan of parameter space, as the asymmetries presented in Figure 2 are from a particular slice only. For that purpose, a more systematic and accurate calculation of the rates of right-handed neutrino production and pair creation and annihilation of doublet leptons, that we estimate here in Appendix A, would be beneficial. The relevant techniques are currently under development, see Refs. [42–46].

The Closed-Time-Path method so far has already proven useful in order to describe the emergence of the baryon asymmetry of the Universe in a more systematic manner

and in order to calculate certain finite-temperature corrections to the approximations that are most commonly applied [8–10, 13, 20–29]. Together with Ref. [14], the present work shows that these methods can also be useful in order to identify sources for the CP -asymmetry, that have not been considered before. The further development of non-equilibrium field theory techniques and their application to the comparably simple problem of Leptogenesis will therefore prove helpful for progress on both, the qualitative and quantitative understanding of the emergence of the baryon-asymmetric Universe.

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A Pair Creation and Annihilation Rates and Flavour Sensitive Rates

Pair creation and annihilation can be mediated either through the s -channel or the t -channel. Individual s -channel processes are typically suppressed compared to t -channel processes by a factor of a few (essentially the logarithm of the squared gauge coupling), because the latter may be infrared enhanced due to the exchange of soft particles. Nonetheless, due to the large number of matter degrees of freedom in the Standard Model, it should be a sufficient approximation to consider s -channel processes for pair creation and annihilation of matter particles and Higgs bosons only. Technically, it should also be possible to account for the t -channel contributions, which would however be more complicated due to the infrared enhancement and its screening.

A standard procedure for calculating cosmological interaction rates of non-relativistic particles relies on the reduced cross section $\hat{\sigma}(s)$, where s is the Mandelstam s -variable. The main simplification arises due to the use of Maxwell statistics. For massless particles, this approximation incurs an order one inaccuracy. Moreover, the t -channel contributions may leave the reduced cross section infrared divergent, a problem that should be accounted for by including the screening effects in the plasma. In conjunction with the comments above, we should expect that the following procedure yields an $\mathcal{O}(1)$ estimate of the weighted pair creation and annihilation rate B_ℓ^g . A more accurate calculation of this quantity may be subject of a future study.

The total annihilation rate of massless particles A into massless matter or Higgs particles B in Maxwell approximation is given by

$$\begin{aligned} \gamma_{A\bar{A} \rightarrow B\bar{B}} &= \int \frac{d^3 k_A}{2|\mathbf{k}_A|} \frac{d^3 k_{\bar{A}}}{2|\mathbf{k}_{\bar{A}}|} \frac{d^3 k_B}{2|\mathbf{k}_B|} \frac{d^3 k_{\bar{B}}}{2|\mathbf{k}_{\bar{B}}|} e^{-|\mathbf{k}_A|/T} e^{-|\mathbf{k}_{\bar{B}}|/T} |\mathcal{M}_{A\bar{A} \rightarrow B\bar{B}}|^2 \\ &= \frac{T}{64\pi^4} \int_0^\infty ds \sqrt{s} K_1 \left(\frac{\sqrt{s}}{T} \right) \hat{\sigma}(s). \end{aligned} \quad (55)$$

The squared matrix element is understood to be a sum over all external polarisation and weak isospin states. The reduced cross section is defined as

$$\hat{\sigma}(s) = \frac{1}{8\pi s} \int_0^\infty dt |\mathcal{M}_{A\bar{A} \rightarrow B\bar{B}}|^2. \quad (56)$$

Let $f_{A,B}$ denote chiral fermions, $b_{A,B}$ bosons. We obtain that

$$\gamma_{f_A \bar{f}_A \rightarrow f_B \bar{f}_B} = G \frac{T^4}{24\pi^5}, \quad (57a)$$

$$\gamma_{f_A \bar{f}_A \rightarrow b_B \bar{b}_B} = G \frac{T^4}{192\pi^5}, \quad (57b)$$

$$\gamma_{b_A \bar{b}_A \rightarrow b_B \bar{b}_B} = G \frac{T^4}{384\pi^5}. \quad (57c)$$

When $f_{X,Y}$ and $b_{X,Y}$ are $SU(2)_L$ doublets, $G = \frac{3}{4}g_2^4 + g_1^4(q_A^2 q_B^2)$, where $q_{A,B}$ denotes the $U(1)_Y$ weak hypercharge of the particles. For singlets, $G = g_1^4(q_A^2 q_B^2)$. For the doublets, we have to divide the rates by two in order to average over the weak isospin such that we obtain the production rate for a single isospin state. Summing over all Standard Model degrees of freedom for B , we obtain

$$\gamma_{\ell\bar{\ell} \rightarrow \text{anything}} = \left(\frac{97}{512\pi^5} g_2^4 + \frac{163}{4608\pi^5} g_1^4 \right) T^4. \quad (58)$$

The averaged rate for a single flavour-sensitive process is therefore [10]

$$\Gamma_{\text{av}}^g = \frac{\gamma_{\ell\bar{\ell} \rightarrow \text{anything}}}{3/(2\pi)^2 \zeta(3) T^3} \approx 4.6 \times 10^{-4} T. \quad (59)$$

We have taken here for the numerical values $g_2 = 0.6$ and $g_1 = 0.4$ at the time of Leptogenesis. Now with the order one approximation that all lepton doublets experience the same reaction rate, regardless of their momentum, we express

$$B_\ell^g n_\ell^+ = \int \frac{d^3 k}{(2\pi)^3} |\mathbf{k}| \Gamma_{\text{av}}^g f_\ell = \delta n_\ell^+ \frac{12\Gamma_{\text{av}}^g}{T^3} \int \frac{d^3 k}{(2\pi)^3} |\mathbf{k}| e^{-|\mathbf{k}|/T}, \quad (60)$$

such that it follows that

$$B_\ell^g = \frac{36}{\pi^2} \Gamma_{\text{av}}^g T = 1.7 \times 10^{-3} T^2. \quad (61)$$

The difference when using a Fermi-Dirac instead of a Maxwell distribution in the above integral (64) is, due to the factor $|\mathbf{k}|$ that favours contributions with large momentum, only about 6%. Using the same methods, we obtain for the weighted annihilation rate of Higgs particles

$$B_\phi^g = 1.0 \times 10^{-4} T^2. \quad (62)$$

As for the flavour sensitive rates, we proceed similarly and make our present estimates as follows: Ref. [46] provides the number for the total production rate for sterile right-handed neutrinos. This gives an approximation for the flavour-sensitive scatterings that involve charged right-handed charged leptons. The main difference is that the charged right-handed leptons may also radiate $U(1)_Y$ gauge bosons, an effect that is therefore neglected when adopting the number $h^\dagger h 5 \times 10^{-4} T^4$ for the total reaction rate from Ref. [46]. The averaged rate for a single flavour-sensitive process then is

$$\Gamma_{\text{av}}^\# = \frac{5 \times 10^{-4}}{3/(2\pi)^2 \zeta(3)} T \approx 3 \times 10^{-3} T. \quad (63)$$

In the approximation of the same reaction rate for all leptons in phase space, we obtain

$$B_\ell^\# h^\dagger h \delta n_\ell^+ = \int \frac{d^3 k}{(2\pi)^3} |\mathbf{k}| \Gamma_{\text{av}}^\# h^\dagger h \delta f_\ell = h^\dagger h \delta n_\ell^+ \frac{12 \Gamma_{\text{av}}^\#}{T^3} \int \frac{d^3 k}{(2\pi)^3} |\mathbf{k}| e^{-|\mathbf{k}|/T}, \quad (64)$$

and therefore

$$B_\ell^\# = \frac{36}{\pi^2} \Gamma_{\text{av}}^\# T = 1.0 \times 10^{-2} T^2. \quad (65)$$

In case a more accurate calculation of the lepton asymmetries is desired in the future, the estimates for the rates made in this Appendix, which should be of $\mathcal{O}(1)$ accuracy, need to be improved. The main challenges are then to account for infrared effects and to abandon the approximations of averaging the reaction rates over phase space.

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